MATH 347 HW 1

due September 11, in class

Homework Guildlines

Obviously, your solutions need to be complete and correct, but to receive full credit your write-up should also satisfy the following:

- All the important logical steps in the proof should be present and fully explained.
- All assumptions should be clearly identified.
- Your solutions should be clear and concise. If a sentence does not further the reader's understanding of the solution then it has no place in your write up.
- Use full and grammatically correct English sentences. Mathematical symbols should be used as a tool to distill complex mathematical relationships into a readable format.

Moreover, in order to obtain full credit for the homework, you must write down, in the very least, an attempt at a solution for each problem.

Problems

1.13, 1.14, 1.32, 1.35, 1.41, 1.46 (You do not have to draw Venn diagrams for 1.41) In addition to the problems listed above, please show the following:

- (1) Show that if $X \subseteq Y$, then $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$.
- (2) Show that

$$\mathcal{P}(X) \cap \mathcal{P}(Y) = \mathcal{P}(X \cap Y)$$

and

$$\mathcal{P}(X) \cup \mathcal{P}(Y) \subseteq \mathcal{P}(X \cup Y)$$

- (3) The complement of \emptyset in a set *X* is just *X*, i.e. that $\emptyset^c = X$. Similarly, show that $X^c = \emptyset$.
- (4) Show the following equality of sets where *A*, *B*, *X*, *Y* are sets:
 - (a) $(A \cup B) \times X = (A \times X) \cup (B \times X)$
 - (b) $(A \cap B) \times (X \cap Y) = (A \times X) \cap (B \times Y)$
 - (c) $(A \setminus B) \times X = (A \times X) \setminus (B \times X)$

(5) Finally, let $f : X \to Y$ be a function. Let A, B be subsets of Y. Show that

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$

and

$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$$

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